Solution to Problem Set 2

Q1:

See figure 1 attached. A and B are on the same indifference curve, so $A\tilde{}B$; B and C are on the same indifference curve, so $B\tilde{}C$. Transitivity implies that $A\tilde{}C$. But C is on the indifference curve which is associated with a higher utility level than the one A is on, that is, $C \succ A$. It contradicts with $A\tilde{}C$, so under standard assumptions indifference curves will never intersect with each other. Continuity, monotonicity and convexity are assumed to rule out bizarre cases.

Q2:

See textbook for a more detailed explanation. In short, with ordinal preference we only care about the ranking of different choices or bundles, you can assign any numbers to those choices as long as the ranking is preserved. For cardinal preference, the numbers you assign may be unique and follow certain fixed rules (for example, some medical machine put on your head to monitor certain fluid flow rate).

Q3:

(1).

$$MRS = -\frac{U_x}{U_y} = -\frac{\theta \alpha x^{\alpha-1} y^{\beta}}{\theta \beta x^{\alpha} y^{\beta-1}} = -\frac{\alpha y}{\beta x}$$

(2).

$$MRS = -\frac{U_x}{U_y} = -\frac{\alpha}{\beta}$$

(3).

$$MRS = -\frac{U_x}{U_y} = -\frac{\alpha}{\beta y^{-1}} = -\frac{\alpha y}{\beta}$$

(4).

$$MRS = -\frac{U_x}{U_y} = -\frac{1+y}{1} = -1 - y$$

(5).

$$MRS = -\frac{U_x}{U_y} = -\frac{\frac{1}{2}(x^2+y^2)^{\frac{1}{2}-1} \times 2x}{\frac{1}{2}(x^2+y^2)^{\frac{1}{2}-1} \times 2y} = -\frac{x}{y}$$

(6).

$$MRS = -\frac{U_x}{U_y} = -\frac{\theta x^{\theta-1}}{\theta y^{\theta-1}} = -\frac{x^{\theta-1}}{y^{\theta-1}} = -(\frac{x}{y})^{\theta-1}$$

Q4:

(1). See figure 2 attached.

(2). See figure 3 attached.

Q5:

(1). See figure 4 attached.

(2). See figure 5 attached.

(3). See figure 6 attached.

Q6:

$$MRS = MRT$$

$\rightarrow$

$$\frac{U_x}{U_y} = \frac{p_x}{p_y}$$

$\rightarrow$

$$\frac{\alpha y}{\beta x} = \frac{p_x}{p_y}$$

$\rightarrow$

$$y = \frac{p_x}{p_y}\frac{\beta}{\alpha}x$$

Plug the above expression of y into the budget constraint, we have,

$$p_x x + p_y y = I$$

$\rightarrow$

$$p_x x + p_y \frac{p_x}{p_y}\frac{\beta}{\alpha}x = I$$

$\rightarrow$

$$p_x x + p_x \frac{\beta}{\alpha}x = I$$

$\rightarrow$

$$p_x x\left(1 + \frac{\beta}{\alpha}\right) = I$$

$\rightarrow$

$$x^* = \frac{I}{\left(1+\frac{\beta}{\alpha}\right)p_x}$$

$\rightarrow$

$$y^* = \frac{p_x}{p_y}\frac{\beta}{\alpha}\frac{I}{\left(1+\frac{\beta}{\alpha}\right)p_x} = \frac{\beta}{\alpha}\frac{I}{\left(1+\frac{\beta}{\alpha}\right)p_y}$$

Q7:

Use the method introduced in class, we only care about two cases here:

case (1).

$$\frac{U_x}{U_y} > \frac{p_x}{p_y}$$

$\rightarrow$

$$1 > \frac{p_x}{p_y}$$

$\rightarrow$

$$\begin{cases} x^* = \frac{I}{p_x} \\ y^* = 0 \end{cases}$$

case (2).

$$\frac{U_x}{U_y} < \frac{p_x}{p_y}$$

$\rightarrow$

$$1 < \frac{p_x}{p_y}$$

$\rightarrow$

$$\begin{cases} x^* = 0 \\ y^* = \frac{I}{p_y} \end{cases}$$

Q8.

Use the method introduced in class. We have $\min(x, 2y)$, so $x = 2y \rightarrow \frac{x}{y} = 2$ which is the ratio we need in constructing the fictitious composite good, z. 1 unit of z includes 2 units of x and 1 unit of y, so the price of z, p_z, is $2p_x + p_y$. The total number of z you can buy is $z^* = \frac{I}{p_z} = \frac{I}{2p_x+p_y}$, so

$$\begin{cases} x^* = 2z^* = \frac{2I}{2p_x+p_y} \\ y^* = z^* = \frac{I}{2p_x+p_y} \end{cases}$$

Figure 1

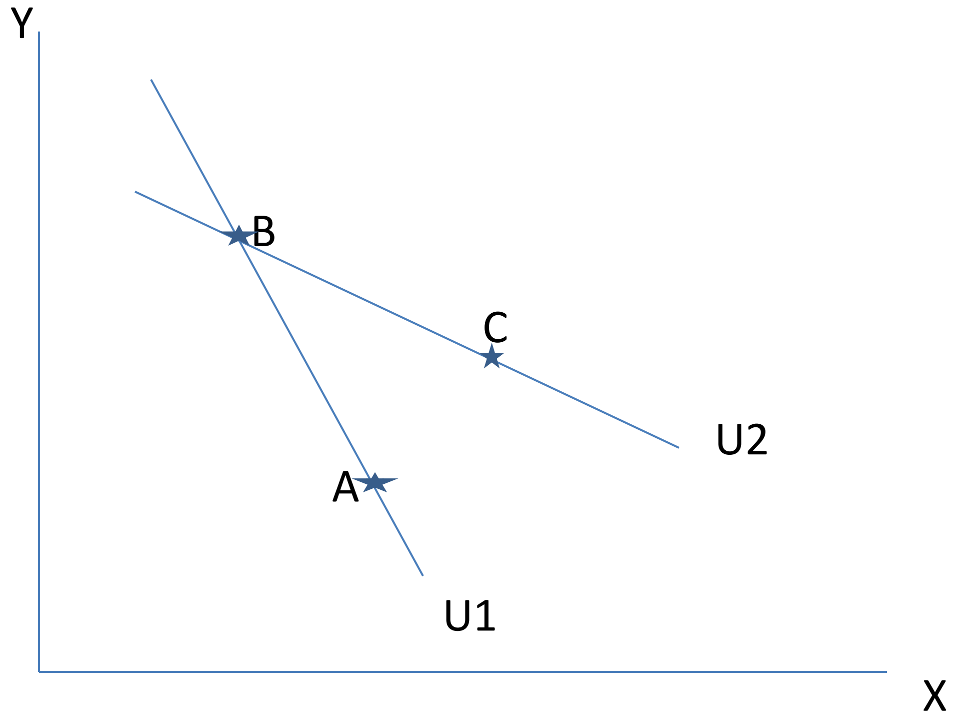

Figure 2

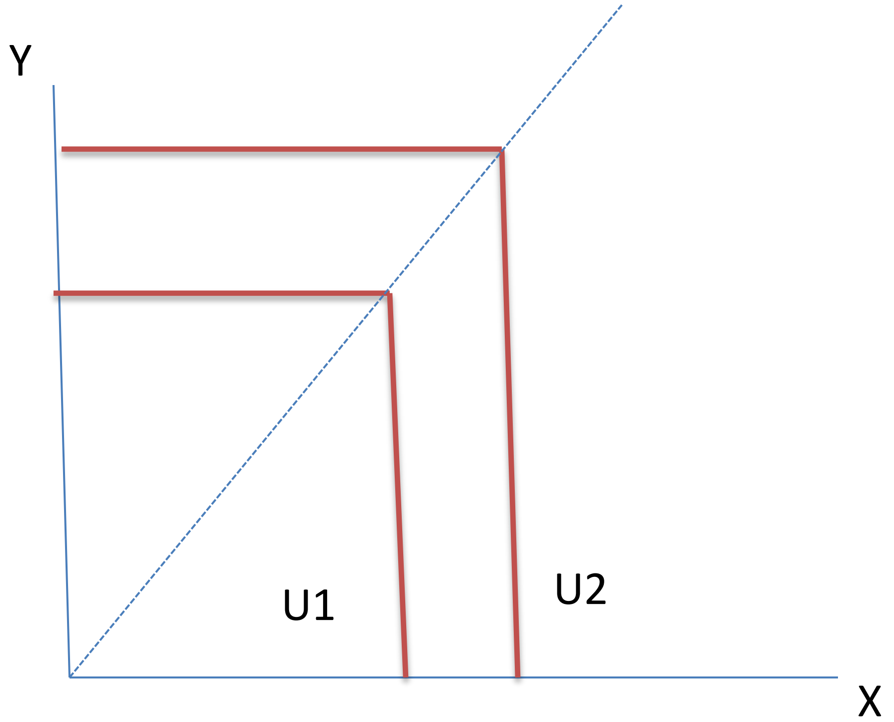

Figure 3

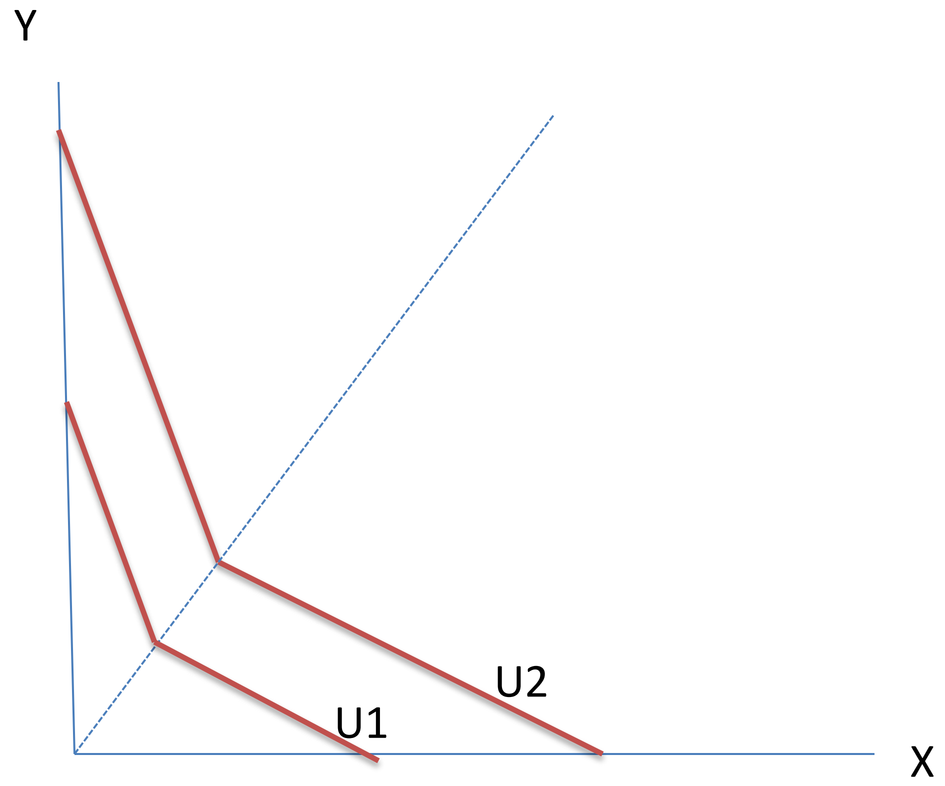

Figure 4

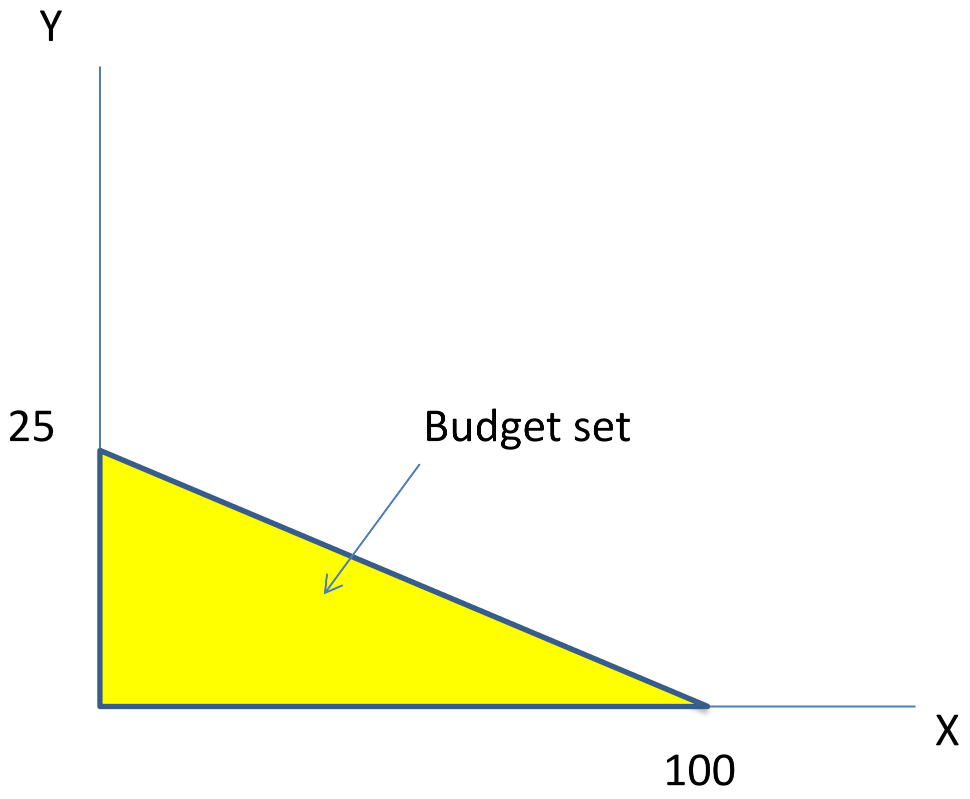

Figure 5

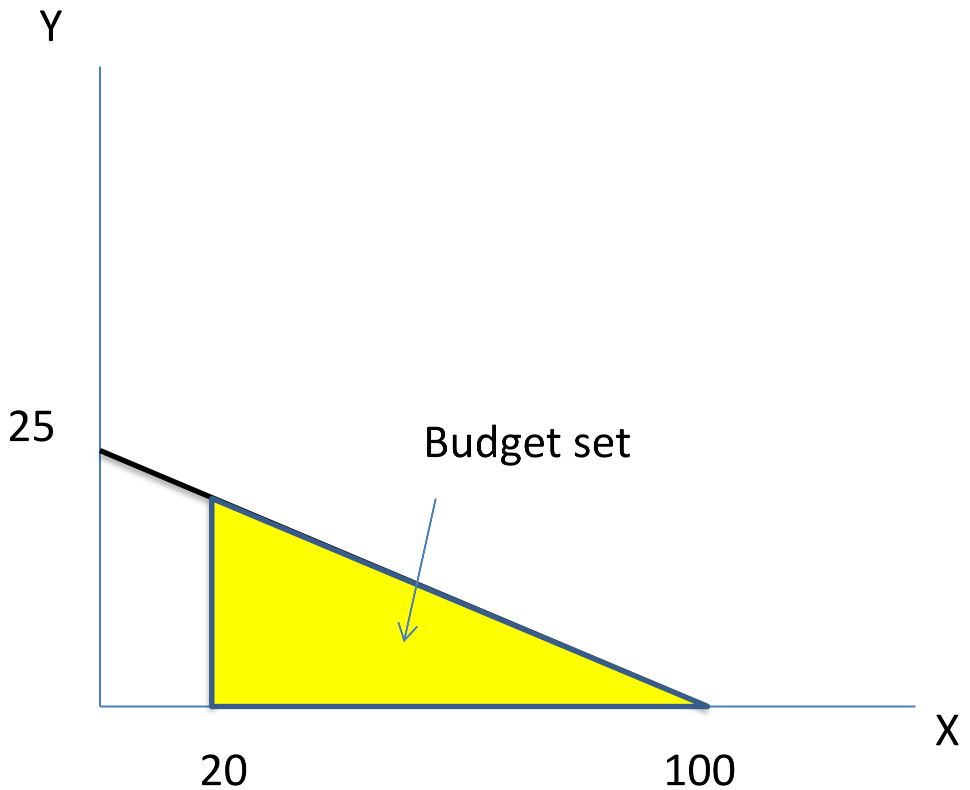

Figure 6