Solution to Problem Set 1

Q1:

(1). $\frac{\partial U}{\partial x} = 8x, \frac{\partial U}{\partial y} = 6y$

(2). $\frac{\partial U}{\partial x} = 8 \times 1 = 8, \frac{\partial U}{\partial y} = 6 \times 2 = 12$

(3). $dU = 8xdx + 6ydy$

(4). $dU = 0 \rightarrow 0 = 8xdx + 6ydy \rightarrow \frac{dy}{dx} = -\frac{4x}{3y}$

(5). $U(1, 2) = 4 \times 1 + 3 \times 2^2 = 16$

Q2:

$$\ln U(x, y, m, n) = \alpha \ln x + \beta \ln y + \frac{1}{\theta} \ln(m^\theta + n^\theta)$$

Q3:

(1). $Q^d = Q^s \rightarrow 120 - 0.2p = 20 + 4.8p \rightarrow p^* = 20, Q^* = 116$

(2). There is shortage!

$$Q^d = 120 - 0.2 \times 15 = 117$$

$$Q^s = 20 + 4.8 \times 15 = 92$$

(3). $Q^d = Q^s \rightarrow 120 - 0.2p = 20 + 9.8p \rightarrow p^* = 10, Q^* = 118$

(4). This question is tricky, note that in this case the price ceiling is set above the equilibrium price in Guelph, so it has no impact on the local housing market! The market price will stay at its equilibrium price, \$10! Market clears, neither surplus nor shortage exists!

$$Q^d = 120 - 0.2 \times 10 = 118$$

$$Q^s = 20 + 9.8 \times 10 = 118$$

Q4:

(1). $Q^d = Q^s \rightarrow 90 - 2p = 30 + 4p \rightarrow p^* = 10, Q^* = 70$

(2). $Q^d = Q^s \rightarrow 150 - 2p = 30 + 4p \rightarrow p^* = 20, Q^* = 110 < 120$

Q5:

(1).

$$\frac{\partial D}{\partial p}\frac{\partial p}{\partial a} + \frac{\partial D}{\partial p}\frac{\partial p}{\partial b} + \frac{\partial D}{\partial a} = \frac{\partial S}{\partial p}\frac{\partial p}{\partial a} + \frac{\partial S}{\partial p}\frac{\partial p}{\partial b} + \frac{\partial S}{\partial b}$$

$$\left(\frac{\partial D}{\partial p} - \frac{\partial S}{\partial p}\right)\frac{\partial p}{\partial a} = \left(\frac{\partial S}{\partial p} - \frac{\partial D}{\partial p}\right)\frac{\partial p}{\partial b} + \frac{\partial S}{\partial b} - \frac{\partial D}{\partial a}$$

$$\frac{\partial p}{\partial a} = -\frac{\partial p}{\partial b} + \frac{\frac{\partial S}{\partial b} - \frac{\partial D}{\partial a}}{\frac{\partial D}{\partial p} - \frac{\partial S}{\partial p}}$$

(2).

$\frac{\partial p}{\partial b} < 0$, $\frac{\partial S}{\partial b} < 0$, $\frac{\partial D}{\partial a} > 0$, $\frac{\partial D}{\partial p} < 0$, $\frac{\partial S}{\partial p} > 0$,we have

$$\frac{\partial p}{\partial a} = -\frac{\partial p}{\partial b} + \frac{\frac{\partial S}{\partial b} - \frac{\partial D}{\partial a}}{\frac{\partial D}{\partial p} - \frac{\partial S}{\partial p}} > 0$$

Q6:

(1).

$$\begin{aligned} \epsilon &= \frac{dD}{dp}\frac{p}{D} \\ &= \alpha y \times (-1) \times p^{-2} \times \frac{p}{D} \\ &= \alpha y \times (-1) \times p^{-1} \times \alpha^{-1} y^{-1} p \\ &= -1 \end{aligned}$$

(2).

$$\begin{aligned} \frac{\partial D}{\partial p_o}\frac{p_o}{D} &= p^{\alpha} \times \beta \times p_o^{\beta-1} \times \frac{p_o}{D} \\ &= p^{\alpha} \times \beta \times p_o^{\beta} \times p^{-\alpha} p_o^{-\beta} \\ &= \beta \end{aligned}$$

Q7:

This is an open question. It may be related to the fact that almost all major movie studios (Warner Brothers, Paramount, Sony, Universal, etc.) in the US produce a variety of movies: popular and unpopular. Movie studios may not like to let the movie theaters single out the potential quality of their movies by charging different prices based on expected popularity. It may also be related to the tradition of movie industry pricing practice.

Question 4: Graph