Solution to Problem Set 6
Econ 301, Summer 2013
Iowa State University

Q1:

$$[\text{Confess, Confess}]$$

Q2:

$$[\text{Stop, Accelerate}], [\text{Cross, Yield}]$$

Q3:

$$[\text{Montreal, Montreal}]$$

Q4:

(1).

$$\pi = PQ - 0 = 150Q - Q^2 \Rightarrow \frac{d\pi}{dQ} = 0 \Rightarrow \begin{cases} Q^m = 75 \\ P^m = 75 \\ \pi^m = 5625 \end{cases}$$

(2).

Two identical firms compete in quantity

$$Q = q_1 + q_2 = 150 - P$$

$$MC_1 = MC_2 = 0$$

Firm 1:

$$\max_{q_1} \pi_1 = Pq_1 - MC_1 q_1 = (P - MC_1)q_1 = (150 - q_1 - q_2 - 0)q_1 = 150q_1 - q_1 q_2 - q_1^2$$

$$\frac{d\pi_1}{dq_1} = 150 - q_2 - 2q_1 = 0 \Rightarrow q_1^* = \frac{150 - q_2}{2}$$

This is the **best response function** of firm 1: his optimal quantity depends on his rival, firm 2's strategy, i.e., how much to produce, q_2**.** This is the **strategic interdependence** introduced in game theory!

Firm 2:

$$\max_{q_2} \pi_2 = Pq_2 - MC_2 q_2 = (P - MC_2)q_2 = (150 - q_1 - q_2 - 0)q_2 = 150q_2 - q_1 q_2 - q_2^2$$

$$\frac{d\pi_2}{dq_2} = 150 - 2q_2 - q_1 = 0 \Rightarrow q_2^* = \frac{150 - q_1}{2}$$

This is the **best response function** of firm 2: his optimal quantity depends on his rival, firm 1's strategy, i.e., how much to produce, q_1**.**

Nash equilibrium: the intersection of the above two best response functions, i.e., $q_1^* = q_2^*$

$$q_1^* = q_2^* \Rightarrow q_2^* = \frac{150 - q_1}{2} = \frac{150 - q_2}{2} \Rightarrow q_1^* = q_2^* = 50$$

So the *Nash equilibrium* is $[q_1^* = 50, q_2^* = 50]$

$$P = 150 - Q = 150 - q_1 - q_2 = 50$$

Profits or payoffs for each firm:

$$\pi_1^* = \pi_2^* = 50 \times 50 = 2500$$

Q5:

(1).

$$Q = q_A + q_B = 500 - 20P$$

$$\begin{cases} MC_A = 10 \\ MC_B = 8 \end{cases}$$

We can see that firm B has competitive advantage in the sense of producing at a strictly lower cost than firm A, so the *Nash equilibrium* for this case is

$$\begin{cases} P_A^* = 10 \\ P_B^* = 10 - \epsilon \end{cases}$$

(2).

where $\epsilon > 0$ is a very small number, for example, a penny, so firm B can sell at a price which is lower than firm A, he is going to take the entire market because firm A can not accept the price which is lower than its marginal cost:

$$\begin{cases} q_A^* = 0 \\ q_B^* = Q = 500 - 20P_B^* = 300 + 20\epsilon \end{cases}$$

Profits or payoffs for each firm:

$$\begin{cases} \pi_A^* = 0 \\ \pi_B^* = (P_B^* - MC_B) \times q_B^* = (10 - \epsilon) \times (300 + 20\epsilon) \approx 3000 \end{cases}$$

Q6:

(1).

Two identical firms compete in quantity, but in a sequential-move setting: firm 1 first decides how much to produce, given firm 1's decision, firm 2 then decides how much to produce.

$$Q = q_1 + q_2 = 130 - P$$

$$MC_1 = MC_2 = 10$$

Solve this game by backward induction:

Firm 2's problem:

$$\max_{q_2} \pi_2 = (P - MC_2)q_2 = (130 - q_1 - q_2 - 10)q_2 = 120q_2 - q_1 q_2 - q_2^2$$

$$\frac{d\pi_2}{dq_2} = 120 - 2q_2 - q_1 = 0 \Rightarrow q_2^* = \frac{120 - q_1}{2}$$

Firm 1's problem, knowing firm 2's best response function:

$$\max_{q_1} \pi_1 = (P - MC_1)q_1 = (130 - q_1 - q_2 - 10)q_1 = (130 - q_1 - \frac{120 - q_1}{2} - 10)q_1 = 60q_1 - \frac{1}{2}q_1^2$$

$$\frac{d\pi_1}{dq_1} = 60 - q_1 = 0 \Rightarrow q_1^* = 60 \Rightarrow q_2^* = \frac{120 - 60}{2} = 30$$

So the *Sub-game perfect equilibrium* (**SPE**) is $[q_1^* = 60, q_2^* = 30]$

(2).

$$P = 130 - Q = 130 - q_1 - q_2 = 40$$

Profits or payoffs for each firm:

$$\begin{cases} \pi_1^* = 60q_1 - \frac{1}{2}q_1^2 = 1800 \\ \pi_2^* = 120q_2 - q_1q_2 - q_2^2 = 900 \end{cases}$$

Q7:

$$\{[L, a], L\}$$

Q8:

(1).

For **G**, **H**, **K** and **J**, clustering coefficient $= \frac{1}{3}$

For **F** and **I**, clustering coefficient $= 0$

For **B** and **D**, clustering coefficient $= \frac{1}{3}$

For **C** and **E**, clustering coefficient $= \frac{3}{10}$

For **A**, clustering coefficient $= \frac{2}{3}$

So the clustering coefficient of this network:

$$CL = \frac{\frac{1}{3} \times 4 + 0 \times 2 + \frac{1}{3} \times 2 + \frac{3}{10} \times 2 + \frac{2}{3}}{11} = 0.297$$

(2).

$$\begin{cases} H^A = \frac{1}{2} = 50\% \\ H^E = \frac{2}{5} < 50\% \\ H^F = 1 > 50\% \\ H^K = \frac{2}{3} > 50\% \end{cases}$$

(3).

F and **K** show **inbreeding homophily** since most of their friends are of their own type.