Solution to Problem Set 5

Econ 301, Summer 2013

Iowa State University

Q1:

(1).

$$R = pQ = 53Q - Q^2$$

$$MR = MC \Rightarrow 53 - 2Q = 5 \Rightarrow Q^m = 24 \Rightarrow p^m = 53 - 24 = 29$$

$$\pi^m = p^m Q^m - 5Q^m = 576$$

(2).

$$p = MR = MC$$

$$p^* = MC = 5$$

$$Q^* = 53 - 5 = 48$$

(3).

$$CS^{competitive} = \frac{1}{2} \times (53 - 5) \times 48 = 1152$$

$$CS^{monopoly} = \frac{1}{2} \times (53 - 29) \times 24 = 288$$

$$CS^{competitive} + \pi^m = 864 < 1152 = CS^{competitive}$$

Q2:

(1).

$$R_1 = P_1 Q_1 = 55Q_1 - Q_1^2$$

$$R_2 = P_2 Q_2 = 35Q_2 - \frac{1}{2}Q_2^2$$

$$MR_1 = MC \Rightarrow 55 - 2Q_1 = 5 \Rightarrow \begin{cases} Q_1^m = 25 \\ P_1^m = 30 \\ \pi_1^m = (30 - 5) \times 25 = 625 \end{cases}$$

$$MR_2 = MC \Rightarrow 35 - Q_2 = 5 \Rightarrow \begin{cases} Q_2^m = 30 \\ P_2^m = 20 \\ \pi_2^m = (20 - 5) \times 30 = 450 \end{cases}$$

$$\pi^m = \pi_1^m + \pi_2^m = 625 + 450 = 1075$$

(2).

$$\begin{cases} P_1 = 55 - Q_1 \\ P_2 = 35 - \frac{1}{2}Q_2 \end{cases}, P_1 = P_2 \Rightarrow 55 - Q_1 = 35 - \frac{1}{2}Q_2 \Rightarrow Q_2 = 2Q_1 - 40$$

$$\begin{aligned} \max_{Q_1} \pi &= P(Q_1 + Q_2) - 5 \times (Q_1 + Q_2) \\ &= (55 - Q_1)(Q_1 + 2Q_1 - 40) - 5 \times (Q_1 + 2Q_1 - 40) \\ &= (55 - Q_1)(3Q_1 - 40) - 5 \times (3Q_1 - 40) \\ &= -3Q_1^2 + 190Q_1 - 2000 \end{aligned}$$

$$\frac{d\pi}{dQ_1} = -6Q_1 + 190 = 0 \Rightarrow \begin{cases} Q_1 = 31.67 \\ Q_2 = 2Q_1 - 40 = 23.34 \\ P = 55 - Q_1 = 23.33 \\ \pi = P(Q_1 + Q_2) - 5(Q_1 + Q_2) = 1008.3 \end{cases}$$

(3).

$$P_2 = P_1 - 5 \Rightarrow 35 - \frac{1}{2}Q_2 = 55 - Q_1 - 5 \Rightarrow Q_2 = 2Q_1 - 30$$

$$\begin{aligned}
\max_{Q_1} \pi &= P_1Q_1 + P_2Q_2 - 5 \times (Q_1 + Q_2) \\
&= P_1Q_1 + (P_1 - 5)Q_2 - 5 \times (Q_1 + Q_2) \\
&= P_1(Q_1 + Q_2) - 5Q_2 - 5 \times (Q_1 + Q_2) \\
&= (P_1 - 5)(Q_1 + Q_2) - 5Q_2 \\
&= (55 - Q_1 - 5)(Q_1 + 2Q_1 - 30) - 5(2Q_1 - 30) \\
&= -3Q_1^2 + 170Q_1 - 1350
\end{aligned}$$

$$\frac{d\pi}{dQ_1} = -6Q_1 + 170 = 0 \Rightarrow \begin{cases} Q_1 = 28.3 \\ Q_2 = 2Q_1 - 30 = 26.6 \\ P_1 = 55 - Q_1 = 26.7 \\ P_2 = P_1 - 5 = 21.7 \\ \pi = P_1Q_1 + P_2Q_2 - 5 \times (Q_1 + Q_2) = 1058.33 \end{cases}$$

(4).

$$P = MC = 5 \Rightarrow \begin{cases} Q_1 = 55 - 5 = 50 \\ Q_2 = 70 - 2 \times 5 = 60 \end{cases} \Rightarrow Q = Q_1 + Q_2 = 110$$

$$\begin{cases} CS_1 = \frac{1}{2} \times (55 - 5) \times 50 = 1250 \\ CS_2 = \frac{1}{2} \times (35 - 5) \times 60 = 900 \end{cases}$$

So there are two possible pricing strategies, serving both types or serving the "rich" only:

$$\begin{cases} P^{rich} = 1250 + 5Q \\ P^{both} = 900 + 5Q \end{cases}$$

if only serve rich type, $Q_2 = 0 \Rightarrow Q = Q_1 + 0 = 50$:

$$P = 1250 + 5Q \Rightarrow \pi = (1250 + 5 \times 50) \times 50 - 5 \times 50 = 74750$$

if serve both types, $Q = Q_1 + Q_2 = 110$:

$$P = 900 + 5Q \Rightarrow \pi = P(Q_1 + Q_2) - 5 \times (Q_1 + Q_2) = (P - 5)(Q_1 + Q_2) = (900 + 5 \times 110 - 5) \times 110 = 158950$$

So it is better for the monopolist to serve both types since it results in higher profits!