Solution to Problem Set 4

Econ 301, Summer 2013

Iowa State University

Q1:

(1).

$$\begin{cases} MP_L = 0.6K^{0.7}L^{-0.7} \\ MP_K = 1.4K^{-0.3}L^{0.3} \end{cases}$$

(2).

$$\begin{cases} MP_L = 2 \\ MP_K = 6 \end{cases}$$

(3). Not Differentiable!

Q2:

(1).

$$\frac{dMP_L}{dL} = -0.42K^{0.7}L^{-1.7} < 0$$

$$\frac{dMP_K}{dK} = -4.2K^{-1.3}L^{0.3} < 0$$

(2).

$$\begin{cases} MP_L = 0.5K^{0.5}L^{-0.5} + 0.4K^{0.2}L^{-0.6} \\ MP_K = 0.5K^{-0.5}L^{0.5} + 0.2K^{-0.8}L^{0.4} \end{cases}$$

$$\frac{dMP_L}{dL} = -0.25K^{0.5}L^{-1.5} - 0.24K^{0.2}L^{-1.6} < 0$$

$$\frac{dMP_K}{dK} = -0.25K^{-1.5}L^{0.5} - 0.16K^{-1.8}L^{0.4} < 0$$

Q3:

(1). See figure 1 attached.

(2). See figure 2 attached.

(3). See figure 3 attached.

Q4:

(1).

$$MRTS = -\frac{0.5K^{0.5}L^{-0.5}}{0.5K^{-0.5}L^{0.5}} = -\frac{K}{L}$$

(2).

$$MRTS = -\frac{2}{6} = -\frac{1}{3}$$

(3).

$$MRTS = -\frac{2(K^{0.5}+L^{0.5})\times 0.5\times L^{-0.5}}{2(K^{0.5}+L^{0.5})\times 0.5\times K^{-0.5}} = -\frac{L^{-0.5}}{K^{-0.5}} = -\left(\frac{L}{K}\right)^{-0.5} = -\left(\frac{K}{L}\right)^{0.5}$$

Q5:

(1).

$$MRTS = -\frac{K}{L} \implies |MRTS| = \frac{K}{L} \implies \ln|MRTS| = \ln\left(\frac{K}{L}\right) \implies \frac{d\ln\left(\frac{K}{L}\right)}{d\ln|MRTS|} = 1 \implies \sigma = 1$$

(2).

$$\sigma = \infty$$

(3).

$$MRTS = -\left(\frac{K}{L}\right)^{0.5} \implies |MRTS| = \left(\frac{K}{L}\right)^{0.5} \implies \ln|MRTS| = 0.5\ln\left(\frac{K}{L}\right) \implies \frac{d\ln\left(\frac{K}{L}\right)}{d\ln|MRTS|} = \frac{1}{0.5} = 2 \implies \sigma = 2$$

Q6:

(1).

$$0.3 + 0.5 = 0.8 < 1 \implies DRS$$

(2).

$$f(2K, 2L) = 2(2K)^{0.6}(2L)^{-0.6} = 2K^{0.6}L^{-0.6} = f(K, L) < 2f(K, L) \implies DRS$$

(3).

$$0.3 + 0.3 + 0.1 = 0.7 < 1 \implies DRS$$

(4).

$$\begin{aligned} f(2K, 2L) &= (2K) + 3(2L)^{0.5} + (2K)^{0.3}(2L)^{0.5} \\ &= 2K + 2^{0.5}(3L^{0.5}) + 2^{0.8}K^{0.3}L^{0.5} \\ &< 2K + 2(3L^{0.5}) + 2K^{0.3}L^{0.5} = 2\left(K + 3L^{0.5} + K^{0.3}L^{0.5}\right) = 2f(K, L) \implies DRS \end{aligned}$$

(5).

$$\begin{aligned} f(2K, 2L) &= \left[(2K)^{0.25} + (2L)^{0.25}\right]^2 (2K)^{0.25}(2L)^{0.25} \\ &= \left[2^{0.25}K^{0.25} + 2^{0.25}L^{0.25}\right]^2 2^{0.25}K^{0.25}2^{0.25}L^{0.25} \\ &= \left[2^{0.25}\left(K^{0.25} + L^{0.25}\right)\right]^2 2^{0.25}2^{0.25}K^{0.25}L^{0.25} \\ &= \left(2^{0.25}\right)^2\left(K^{0.25} + L^{0.25}\right)^2 2^{0.5}K^{0.25}L^{0.25} \\ &= 2^{0.5}\left(K^{0.25} + L^{0.25}\right)^2 2^{0.5}K^{0.25}L^{0.25} \\ &= 2^{0.5}2^{0.5}\left(K^{0.25} + L^{0.25}\right)^2 K^{0.25}L^{0.25} \\ &= 2\left(K^{0.25} + L^{0.25}\right)^2 K^{0.25}L^{0.25} \\ &= 2f(K, L) \implies CRS \end{aligned}$$

Q7:

(1).

$$|MRTS| = \frac{w}{r} \implies \frac{2 \times 0.4K^{0.6}L^{-0.4}}{2 \times 0.6K^{-0.4}L^{0.6}} = \frac{1}{4} \implies \begin{cases} \frac{K}{L} = \frac{3}{8} \\ 2K^{0.6}L^{0.4} = 100 \end{cases} \implies \begin{cases} K^* = 33.774 \\ L^* = 90.064 \end{cases}$$

$$C(K^*, L^*) = rK^* + wL^* = 4 \times 33.774 + 1 \times 90.064 = 225.16$$

(2).

$$|MRTS| = \frac{36}{12} = 3 > \frac{1}{4} = \frac{w}{r} \implies \frac{MP_L}{w} > \frac{MP_K}{r} \implies \begin{cases} K^* = 0 \\ L^* = \frac{100}{36} = 2.78 \end{cases}$$

$$C(K^*, L^*) = rK^* + wL^* = 4 \times 0 + 1 \times 2.78 = 2.78$$

FIGURE 1

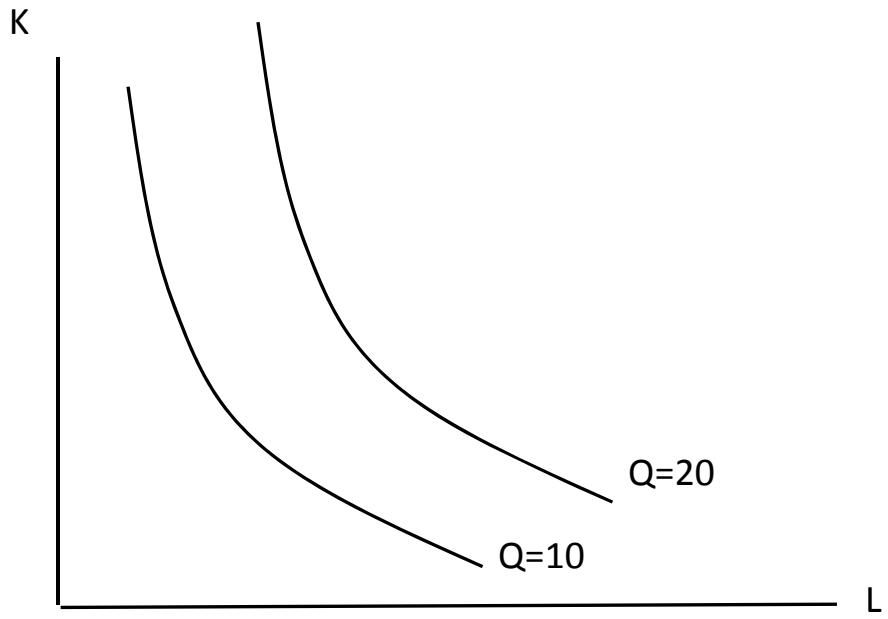

FIGURE 2

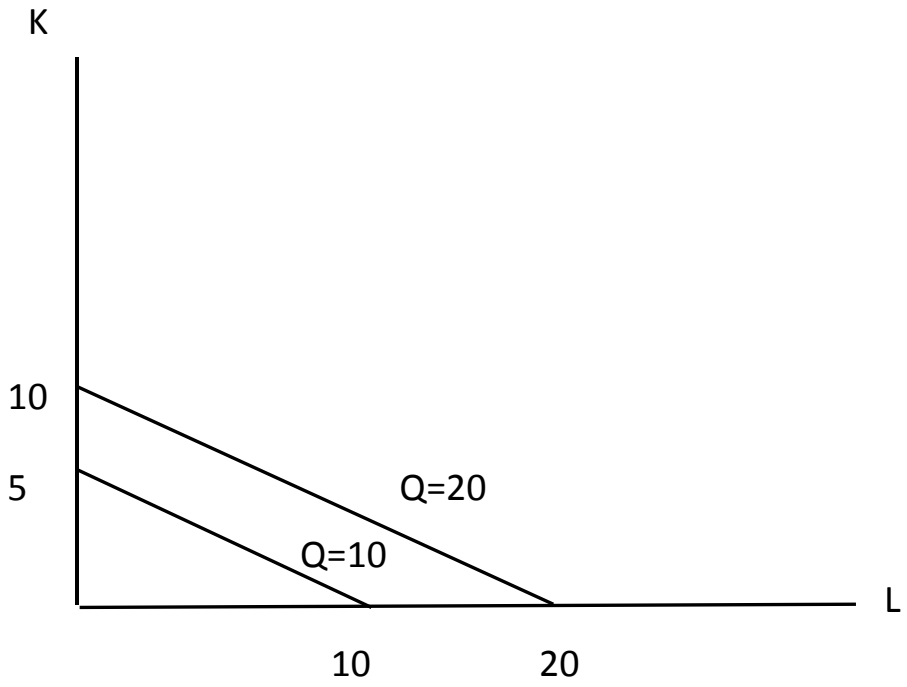

FIGURE 3