ECON301 Summer 2013
Iowa State University
Solution to Problem Set 3

Q1.

(1).

$$MRS = MRT$$

$\Longrightarrow$

$$\frac{\frac{1}{3}x^{-2/3}y^{2/3}}{\frac{2}{3}x^{1/3}y^{-1/3}} = \frac{1}{2}$$

$\Longrightarrow$

$$y = x$$

$\Longrightarrow$

$$U(x,y) = x^c = y^c = 30$$

(2).

$$MRS = 6 > \frac{1}{2} = MRT$$

$\Longrightarrow$

$$\frac{U_x}{U_y} > \frac{p_x}{p_y}$$

$\Longrightarrow$

Corner solution, $y = 0$. so $U(x,y) = 6x + y = 6x$,

$$\begin{cases} x^c = \frac{30}{6} = 5 \\ y^c = 0 \end{cases}$$

(3).

$$U(x,y) = \min(3x, y) = 60 \Longrightarrow 3x = y = 60$$

$\Longrightarrow$

$$\begin{cases} x^c = 20 \\ y^c = 60 \end{cases}$$

Q2.

(1).

$$E(x) = 1 * 0.2 + 2 * 0.3 + 3 * 0.1 + 4 * 0.4 = 2.7$$

(2).

$$V(x) = 0.2 * (1 - 2.7)^2 + 0.3 * (2 - 2.7)^2 + 0.1 * (3 - 2.7)^2 + 0.4 * (4 - 2.7)^2 = 1.41$$

(3).

$$\sqrt{V(x)} = 1.187434$$

(4).

$$EU = \ln 1 * 0.2 + \ln 2 * 0.3 + \ln 3 * 0.1 + \ln 4 * 0.4 = 0.872$$

$$\rho(x) = -\frac{-x^{-2}}{x^{-1}} = \frac{1}{x} > 0$$

Risk-averse!

(5).

$$EU = 1^4 * 0.2 + 2^4 * 0.3 + 3^4 * 0.1 + 4^4 * 0.4 = 115.5$$

$$\rho(x) = -\frac{12x^2}{4x^3} < 0$$

Risk-loving!

(6).

$$EU = (8*1+2)*0.2 + (8*2+2)*0.3 + (8*3+2)*0.1 + (8*4+2)*0.4 = 23.6$$

$$\rho(x) = -\frac{0}{8} = 0$$

Risk-neutral!

Q3.

(1).

$$U(100-\theta) = 0.6*U(20) + 0.4*U(100)$$

$\Longrightarrow$

$$\ln(100-\theta) = 0.6*\ln 20 + 0.4*\ln 100$$

$\Longrightarrow$

$$\theta = \$61.93$$

(2).

$$U(100-\theta) = 0.6*U(20) + 0.4*U(100)$$

$\Longrightarrow$

$$(100-\theta)^4 = 0.6*20^4 + 0.4*100^4$$

$\Longrightarrow$

$$\theta = \$20.43$$

(3).

$$U(100-\theta) = 0.6*U(20) + 0.4*U(100)$$

$\Longrightarrow$

$$100-\theta = 0.6*20 + 0.4*100$$

$\Longrightarrow$

$$\theta = \$48$$

Q4.

(1).

$$\rho(x) = -\frac{-x^{-2}}{x^{-1}} = \frac{1}{x}$$

$$r(x) = x\rho(x) = 1$$

(2).

$$\rho(x) = -\frac{-\theta x^{-\theta-1}}{x^{-\theta}} = \frac{\theta}{x}$$

$$r(x) = x\rho(x) = \theta$$

(3).

$$\rho(x) = -\frac{-\frac{1}{\theta}\exp(-\frac{x}{\theta})}{\exp(-\frac{x}{\theta})} = \frac{1}{\theta}$$

$$r(x) = x\rho(x) = \frac{x}{\theta}$$

Q5.

$$U(x) = E(x) - \frac{1}{2}V(x) = \theta E(A) + (1-\theta)E(B) - \frac{1}{2}\left[\theta^2 V(A) + (1-\theta)^2 V(B)\right]$$

$\Longrightarrow$

$$\frac{dU}{d\theta} = 0$$

$\Longrightarrow$

$$E(A) - E(B) - \theta V(A) + (1-\theta)V(B) = 0$$

$\Longrightarrow$

$$8 - 2 - \theta * 25 + (1-\theta) * 4 = 0$$

$\Longrightarrow$

$$\theta = \frac{10}{29} \approx 0.345 = 34.5\%$$

So invest 34.5% of total wealth in stock A and 65.5% in stock B.

Q6.

We can tell the difference by comparing income elasticities, η.

$$\begin{cases} \eta \geq 0 & \text{normal good} \\ \eta < 0 & \text{inferior good} \\ 0 \leq \eta \leq 1 & \text{necessary good} \\ \eta > 1 & \text{luxuary good} \end{cases}$$

Q7.

$$p_1Q_1 + p_2Q_2 + p_3Q_3 + p_4Q_4 + p_5Q_5 = Y$$

$\Longrightarrow$

$$p_1\frac{dQ_1}{dY} + p_2\frac{dQ_2}{dY} + p_3\frac{dQ_3}{dY} + p_4\frac{dQ_4}{dY} + p_5\frac{dQ_5}{dY} = 1$$

$\Longrightarrow$

$$p_1\frac{dQ_1}{dY}\frac{Y}{Q_1}\frac{Q_1}{Y} + p_2\frac{dQ_2}{dY}\frac{Y}{Q_2}\frac{Q_2}{Y} + p_3\frac{dQ_3}{dY}\frac{Y}{Q_3}\frac{Q_3}{Y} + p_4\frac{dQ_4}{dY}\frac{Y}{Q_4}\frac{Q_4}{Y} + p_5\frac{dQ_5}{dY}\frac{Y}{Q_5}\frac{Q_5}{Y} = 1$$

$\Longrightarrow$

$$\frac{dQ_1}{dY}\frac{Y}{Q_1}\frac{p_1Q_1}{Y} + \frac{dQ_2}{dY}\frac{Y}{Q_2}\frac{p_2Q_2}{Y} + \frac{dQ_3}{dY}\frac{Y}{Q_3}\frac{p_3Q_3}{Y} + \frac{dQ_4}{dY}\frac{Y}{Q_4}\frac{p_4Q_4}{Y} + \frac{dQ_5}{dY}\frac{Y}{Q_5}\frac{p_5Q_5}{Y} = 1$$

$\Longrightarrow$

$$\eta_1\theta_1 + \eta_2\theta_2 + \eta_3\theta_3 + \eta_4\theta_4 + \eta_5\theta_5 = 1$$

where η is the income elasticity and θ is the expenditure share on that good. We know that θ must be non-negative, so it is impossible for all η to be negative otherwise the above equality is violated. So at least one good must be normal!

Q8.

(1).

$$\int_0^1 x dx = \frac{1}{2}x^2 \,|_0^1 = \frac{1}{2} * 1^2 - \frac{1}{2} * 0^2 = \frac{1}{2}$$

(2).

$$\int_2^3 x^{-1} dx = \ln x \,|_2^3 = \ln 3 - \ln 2 = 0.4054651$$

(3).

$$\int_0^1 (x^2 + 2x)dx = \left(\frac{1}{3}x^3 + x^2\right)|_0^1 = \frac{1}{3} * 1^3 + 1^2 - \frac{1}{3} * 0^3 - 0^2 = \frac{4}{3}$$

(4).

$$\int_1^2 \exp(3x)dx = \frac{1}{3}\exp(3x)\,|_1^2 = \frac{1}{3}\exp(3 * 2) - \frac{1}{3}\exp(3 * 1) = 127.7811$$