Due: Thursday, July 11 (in class)

*Challenging questions are marked with *.*

Question 1: Suppose that you know for two goods, x and y, $p_x = 1$ and $p_y = 2$, the target utility level is 30, find the **compensated demand** of x and y for the following utility functions:

1. $U(x, y) = x^{\frac{1}{3}} y^{\frac{2}{3}}$
2. $U(x, y) = 6x + y$
3. $U(x, y) = \min(3x, y)$

Question 2: You are given the following discrete distribution of a random variable, x:

x	1	2	3	4
prob(x)	0.2	0.5	0.1	0.4

1. find the expectation of x.
2. find the variance of x.
3. find the standard deviation of x.
4. what is the expected utility if the utility function is $U(x) = \ln x$? Is the consumer with this utility function risk-averse, risk-neutral or risk-loving? Prove your statement.
5. what is the expected utility if the utility function is $U(x) = x^4$? Is the consumer with this utility function risk-averse, risk-neutral or risk-loving? Prove your statement.
6. what is the expected utility if the utility function is $U(x) = 8x + 2$? Is the consumer with this utility function risk-averse, risk-neutral or risk-loving? Prove your statement.

Question 3: Suppose you know that a consumer has initial wealth, $W = \$100$. There are two possible states of the world, with 40% chance that nothing bad will happen to his money while 60% chance that he would lose \$80. For the following utility functions, find the maximum amount of insurance this consumer is willing to pay in order to fully insure his wealth:

1. $U(x) = \ln x$
2. $U(x) = x^4$
3. $U(x) = x$

Question 4: Find the **absolute** and **relative** Arrow-Pratt measure of risk-aversion for the following utility functions:

1. $U(x) = \ln x$
2. $U(x) = \frac{x^{1-\theta}}{1-\theta}, \theta > 1$
3. $U(x) = -\theta \exp(-\frac{x}{\theta})$

Question 5: Suppose that you are deciding how to design your investment portfolio which involves two stocks, A and B. You have the following statistics available, $E(A) = 8, E(B) = 2, V(A) = 25, V(B) = 4$, and $COV(A, B) = 0$. Assume that you use mean–variance utility function, $U(x) = E(x) - \frac{1}{2}V(x)$, to represent your preference, find the optimal portfolio weights for A and B.

Question 6: What is the difference between normal good and inferior good? What is the difference between necessary good and luxury good?

Question 7: Suppose that you have this budget constraint for five goods, $p_1Q_1 + p_2Q_2 + p_3Q_3 + p_4Q_4 + p_5Q_5 = Y$. Y is total income. Prove that at least one among the five goods must be normal good.

Question 8: Evaluate the following definite integrals:

1. $\int_0^1 x dx$
2. $\int_2^3 x^{-1} dx$
3. $\int_0^1 (x^2 + 2x) dx$
4. $\int_1^2 \exp(3x) dx$