Final Exam: Part I

You have 60 minutes for this part. There are six questions and they are worth 50 points.

Question 1: [7 points]   Suppose that you have this utility function, $U(x,y) = 9\min(x,y) + 3\max(x,y)$, the price for good x is 1 and the price for good y is 2. Total income is 100. Find the demand of x and y which maximize the utility function.

Question 2: [13 points]   You are given the following discrete distribution of a random variable, x:

x	5	1	8	4	10
prob(x)	0.2	0.2	0.3	0.1	0.2

1. find the expectation of x.

2. find the variance and standard deviation of x.

3. what is the expected utility if the utility function is $U(x) = \sqrt{\ln x}$? Is the consumer with this utility function risk-averse, risk-neutral or risk-loving? Prove your statement.

4. what is the expected utility if the utility function is $U(x) = (\ln x)^2$? Is the consumer with this utility function risk-averse, risk-neutral or risk-loving? Prove your statement.

Question 3: [6 points]   **[Bertrand Model]** Suppose firms A and B operate under conditions of constant average and marginal costs, but that $MC_A = 20$, $MC_B = 30$. The market demand for the firms' output is given by $Q = 1000 - 10P$.

1. If the firms practice Bertrand competition, what will be the market price under a Nash equilibrium?

2. What will be the profits for each firm?

Question 4: [10 points]   **[Stackelberg Model]** There are two competing firms selling a homogeneous good and we know the average cost and marginal cost of producing each unit of the good for these two firms: $AC_1 = MC_1 = 10$ and $AC_2 = MC_2 = 10$. Firm 1 moves first and decides on the quantity to sell: q_1; firm 2 moves next and after seeing q_1, decides on the quantity to sell: q_2. $Q = q_1 + q_2$ is the total market demand. Both firms seek to maximize profits and the market demand is: $Q = 200 - P$.

1. What will be the profits for each firm?

2. What will be the profits for each firm if they play **Cournot game** instead of Stackelberg?

Question 5: [6 points]

1. What is the difference between **English auction** and **Dutch auction**?

2. What is the Nash equilibrium like in a **second-price sealed-bid** auction? What is the Nash equilibrium like in a **first-price sealed-bid** auction?

Question 6: [8 points]

1. Suppose that you have this demand function, $D(p) = p^{0.6}m^{0.3}n^{0.5}z^{0.2}$, and you know $p = m = n = z = 71$. What is the **price elasticity of demand**?

2. Suppose that you have this demand function, $D(p) = p^{0.5} + p_o^{0.5}$, and you know $p = 4$ and $p_o = 9$. What is the **cross–price elasticity of demand**?

3. If you know that the income elasticity of demand for HD TV in the Ames area market is 1.2, how do you interpret this number? What does income elasticity mean?