Solution to EXAM 3

Econ 301, Summer 2013

Iowa State University

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Q1:

(1).

$$\sigma = \infty$$

(2).

$$MRTS = -(\frac{K}{L})^{3/4} \Rightarrow \ln|MRTS| = \frac{3}{4}\ln(\frac{K}{L}) \Rightarrow \sigma = \frac{4}{3}$$

Q2:

(1).

$$MRTS = \frac{w}{r} \Rightarrow L = 4K \Rightarrow \begin{cases} K = \frac{Q}{4} \\ L = Q \\ C = 2Q \end{cases}$$

(2).

$$MRTS > \frac{w}{r} \Rightarrow K = 0 \Rightarrow \begin{cases} K = 0 \\ L = \frac{Q}{2} \\ C = \frac{Q}{2} \end{cases}$$

Q3:

(1).

$$0.6 + 0.6 > 1 \Rightarrow IRS$$

(2).

$$0.3 + 0.2 + 0.1 < 1 \Rightarrow DRS$$

(3).

$$\begin{aligned} f(2K,2L) &= \left[(2K)^{0.25} + (2L)^{0.25}\right]^2 (2K)^{0.25}(2L)^{0.25} \\ &= \left[2^{0.25}K^{0.25} + 2^{0.25}L^{0.25}\right]^2 2^{0.25}K^{0.25}2^{0.25}L^{0.25} \\ &= \left[2^{0.25}\left(K^{0.25} + L^{0.25}\right)\right]^2 2^{0.25}2^{0.25}K^{0.25}L^{0.25} \\ &= \left(2^{0.25}\right)^2\left(K^{0.25} + L^{0.25}\right)^2 2^{0.5}K^{0.25}L^{0.25} \\ &= 2^{0.5}\left(K^{0.25} + L^{0.25}\right)^2 2^{0.5}K^{0.25}L^{0.25} \\ &= 2^{0.5}2^{0.5}\left(K^{0.25} + L^{0.25}\right)^2 K^{0.25}L^{0.25} \\ &= 2\left(K^{0.25} + L^{0.25}\right)^2 K^{0.25}L^{0.25} \\ &= 2f(K,L) \implies CRS \end{aligned}$$

Q4:

$$\begin{aligned} f(2K,2L) &= 2f(K,L) \Rightarrow 2(\beta_0 + \beta_1\sqrt{KL} + \beta_2 K + \beta_3 L) = \beta_0 + \beta_1\sqrt{2K2L} + \beta_2 2K + \beta_3 2L \\ &= \beta_0 + 2\beta_1\sqrt{KL} + 2\beta_2 K + 2\beta_3 L \Rightarrow \beta_0 = 0 \end{aligned}$$

Q5:

(1).

$$P = MC \Rightarrow P^* = 10 \Rightarrow Q^* = 500 \Rightarrow CS^* = 2500$$

$$MR = MC \Rightarrow 20 - \frac{1}{25}Q = 12 \Rightarrow \begin{cases} P^m = 16 \\ Q^m = 200 \\ CS^m = 400 \end{cases}$$

(2),

$$loss = 2500 - 400 = 2100$$

Q6:

(1). First-degree price discrimination requires the monopolist has the full information about the market he serves. The monopolist knows exactly each consumer's willingness to pay so he charges each customer the price which equals his or her willingness to pay. There is no consumer surplus when first-degree price discrimination is possible. There is no deadweight loss either because the monopolist has taken all surplus from consumers.

(2). In second-degree price discrimination, the monopolist knows the number of possible types of consumers but he can not tell type A from type B, so he may employ a two-part tariff scheme to make consumers self-reveal themselves. Third-degree price discrimination is related to international or regional marketing, the monopolist know that people from different market have different preferences which can be reflected in their demand functions or curves. The monopolist can take advantage of this information can charge different prices in different market according to consumer preferences in each market.

(3).

$$R_1 = P_1Q_1 = 55Q_1 - Q_1^2$$

$$R_2 = P_2Q_2 = 35Q_2 - \frac{1}{2}Q_2^2$$

$$MR_1 = MC \Rightarrow 55 - 2Q_1 = 15 \Rightarrow \begin{cases} Q_1^m = 20 \\ P_1^m = 35 \\ \pi_1^m = 400 \end{cases}$$

$$MR_2 = MC \Rightarrow 35 - Q_2 = 15 \Rightarrow \begin{cases} Q_2^m = 20 \\ P_2^m = 25 \\ \pi_2^m = 200 \end{cases}$$

$$\pi^m = \pi_1^m + \pi_2^m = 600$$

(4).

$$\begin{cases} P_1 = 55 - Q_1 \\ P_2 = 35 - \frac{1}{2}Q_2 \end{cases}, P_1 = P_2 \Rightarrow 55 - Q_1 = 35 - \frac{1}{2}Q_2 \Rightarrow Q_2 = 2Q_1 - 40$$

$$\begin{aligned} \max_{Q_1} \pi &= P(Q_1 + Q_2) - 15 \times (Q_1 + Q_2) \\ &= -3Q_1^2 + 160Q_1 - 1600 \end{aligned}$$

$$\frac{d\pi}{dQ_1} = -6Q_1 + 160 = 0 \Rightarrow \begin{cases} Q_1 = 26.67 \\ Q_2 = 13.34 \\ P = 28.33 \\ \pi = 533.3 \end{cases}$$

(5).

Note that only arbitrage from market 2 to market 1 is relevant because the price in market 1 is much higher than that in market 2.

$$P_2 = P_1 - 4 \Rightarrow 35 - \frac{1}{2}Q_2 = 55 - Q_1 - 4 \Rightarrow Q_2 = 2Q_1 - 32$$

$$\begin{aligned} \max_{Q_1} \pi &= P_1 Q_1 + P_2 Q_2 - 15 \times (Q_1 + Q_2) \\ &= -3Q_1^2 + 144Q_1 - 1152 \end{aligned}$$

$$\frac{d\pi}{dQ_1} = -6Q_1 + 144 = 0 \Rightarrow \begin{cases} Q_1 = 24 \\ Q_2 = 16 \\ P_1 = 31 \\ P_2 = 27 \\ \pi = 576 \end{cases}$$