Exam 3

You have 60 minutes for this exam. There are six questions in this exam.

Question 1: [10 points]   Find the elasticity of substitution for the following production functions:

1. $f(K, L) = 3K + 18L + 100$

2. $f(K, L) = (K^{0.25} + L^{0.25})^4$

Question 2: [12 points]   Suppose you know that $w = 1, r = 4, Q = \bar{Q}$, find the input demand for capital K and labor L, and the **cost function** for the following production functions:

1. $f(K, L) = 2K^{0.5}L^{0.5}$

2. $f(K, L) = K + 2L$

Question 3: [15 points]   Check the following production functions to see if they are increasing returns to scale (**IRS**), constant returns to scale (**CRS**)or decreasing returns to scale (**DRS**). Prove your statement.

1. $f(K, U) = 2K^{0.6}L^{-0.6}U^{0.6}$

2. $f(K, X, Q) = K^{0.3}H^{0.4}U^{0.3}L^{0.9}M^{0.5}X^{0.2}Q^{0.1}Z^{0.8}$

3. $f(K, L) = (K^{0.25} + L^{0.25})^2 K^{0.25}L^{0.25}$

Question 4: [8 points]   Consider a generalization of the production function, $f(K, L) = \beta_0 + \beta_1\sqrt{KL} + \beta_2 K + \beta_3 L$, where $0 \le \beta_i \le 1, i = 0, 1, 2, 3$. If this function is to exhibit constant returns to scale, what restrictions should be placed on the parameters, $\beta_0, \beta_1, \beta_2, \beta_3$?

Question 5: [15 points]   Suppose a perfectly competitive industry can produce widgets at a constant marginal cost of \$10 per unit. Monopolized marginal costs rise to \$12 per unit because \$2 per unit must be paid to lobbyists to retain the monopolist's market power. Suppose the market demand for widgets is given by $Q = 1000 - 50P$:

1. Calculate the perfectly competitive and monopoly outputs and prices.

2. Calculate the total loss of consumer surplus from monopolization of widget production.

Question 6: [40 points]   Suppose that a monopoly can produce any level of output it wishes at a constant marginal and average cost of \$15 per unit. Assume the monopoly sells its goods in two different markets separated by some distance. The demand curve in the first market is given by $Q_1 = 55 - p_1$, and the demand curve in the second market is given by $Q_2 = 70 - 2p_2$.

1. what is first-degree price discrimination? What are the characteristics of first-degree price discrimination?

2. what is the difference between second-degree price discrimination and third-degree price discrimination?

3. If the monopolist can maintain the separation between the two markets, what level of output should be produced in each market, and what price will prevail in each market? What are total profits in this situation?

4. How would your answer change if the firm was forced to follow a single-price policy?

5. Because of different trade regulation and labor cost, the transportation cost (shipping cost) is asymmetric between market 1 and market 2: it costs \$2 to ship from market 1 to market 2 but it costs \$4 to ship from market 2 to market 1. How would your answer change in this case? What would be the monopolist's new profit level in this situation?

6. Suppose that market 1 (USA) is the monopolist's home market, it can produce any level of output it wishes at a constant marginal and average cost of \$15 per unit. But now if he wants to sell in market 2 (JAPAN) by third-degree price discrimination, he has to produce locally, that is, he has to produce the amount he sells to Japan in Japan (market 2). The cost function is $COST = Q^2$ if the monopolist produces in Japan (market 2). what level of output should be produced in each market, and what price will prevail in each market? What are total profits in this situation?

7. Suppose the firm adopts a linear two-part tariff pricing policy to implement second-degree price discrimination, should the firm serve both markets? If it is better to serve only one market, which one? Show your work.