Solution to EXAM 2

Econ 301, Summer 2013

Iowa State University

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Q1:

(1).

$$MRS = MRT \Longrightarrow \frac{\frac{1}{4}x^{-3/4}y^{3/4}}{\frac{3}{4}x^{1/4}y^{3/4}} = \frac{1}{2} \Longrightarrow \frac{y}{x} = \frac{3}{2} \Longrightarrow \begin{cases} y = \frac{3}{2}x \\ x^{\frac{1}{4}}y^{\frac{3}{4}} = 30 \end{cases} \Longrightarrow \begin{cases} x^c = 22.13 \\ y^c = 33.2 \end{cases}$$

(2).

$$MRS = \frac{1}{3} < \frac{1}{2} = MRT \Longrightarrow \frac{MU_x}{P_x} < \frac{MU_y}{P_y} \Longrightarrow \begin{cases} x^c = 0 \\ y^c = \frac{30}{6} = 5 \end{cases}$$

(3).

$$\overline{U} = \min(6x, 5y) = 30 \Longrightarrow 6x = 5y = 30 \Longrightarrow \begin{cases} x^c = 5 \\ y^c = 6 \end{cases}$$

Q2:

(1).

$$E(X) = 4.7$$

(2).

$$V(X) = 8.61$$

(3).

$$\sqrt{V(X)} = 2.93$$

(4).

$$EU = 2.58$$

$$\rho(x) = -\frac{-2x^{-2}}{2x^{-1}} = \frac{1}{x} > 0, \text{ RISK-AVERSE}$$

(5).

$$EU = 40.1$$

$$\rho(x) = -\frac{2}{2x+2} < 0, \text{ RISK-LOVING}$$

Q3:

$$\max_{\theta} \theta EA + (1-\theta)EB - \frac{1}{2}\left[\theta^2 VA + (1-\theta)^2 VB\right]$$

First-order condition implies:

$$EA - EB - \frac{1}{2}\left[2\theta VA - 2(1-\theta)VB\right] = 0$$

$$\theta = \frac{EA - EB + VA}{VA + VB} = 0.538 = 53.8\%$$

$$1 - \theta = 46.2\%$$

So invest53.8% in stock A and 46.2% in stock B.

Q4:

$$\begin{cases} \theta_x = \frac{1\times 20}{100} = 0.2 \\ \theta_y = \frac{8\times 5}{100} = 0.4 \\ \theta_z = \frac{2\times 20}{100} = 0.4 \end{cases}$$

$$\eta_x\theta_x + \eta_y\theta_y + \eta_z\theta_z = 1 \Longrightarrow 0.5 \times 0.2 + 5 \times 0.4 + \eta_z \times 0.4 = 1 \Longrightarrow \eta_z = -2.75$$

$$\begin{cases} \textbf{Luxuray}: \text{ Y} \\ \textbf{Inferior}: \text{ Z} \\ \textbf{Normal}: \text{X, Y} \end{cases}$$

Q5:

$$x^c(p_x, p_y, \overline{U}) = x^*(p_x, p_y, I) = x^*(p_x, p_y, E)$$

$\Longrightarrow$

$$\frac{\partial x^c}{\partial p_x} = \frac{\partial x^*}{\partial p_x} + \frac{\partial x^*}{\partial E}\frac{\partial E}{\partial p_x} = \frac{\partial x^*}{\partial p_x} + \frac{\partial x^*}{\partial E}x^c$$

$\Longrightarrow$

$$\frac{\partial x^*}{\partial p_x} = \frac{\partial x^c}{\partial p_x} - x^c\frac{\partial x^*}{\partial E}$$

$\Longrightarrow$

$$\frac{\partial x^*}{\partial p_x}\frac{p_x}{x^*} = \frac{\partial x^c}{\partial p_x}\frac{p_x}{x^c} - x^c\frac{p_x}{x^c}\frac{E}{E}\frac{\partial x^*}{\partial E}$$

$\Longrightarrow$

$$\frac{\partial x^*}{\partial p_x}\frac{p_x}{x^*} = \frac{\partial x^c}{\partial p_x}\frac{p_x}{x^c} - \frac{x^c p_x}{E}\frac{E}{x^c}\frac{\partial x^*}{\partial E}$$

$\Longrightarrow$

$$\epsilon_x = \epsilon_x^c - \eta_x\theta_x$$

where

$$\begin{cases} \epsilon_x = \frac{\partial x^*}{\partial p_x}\frac{p_x}{x^*} \\ \epsilon_x^c = \frac{\partial x^c}{\partial p_x}\frac{p_x}{x^c} \\ \eta_x = \frac{E}{x^c}\frac{\partial x^*}{\partial E} \\ \theta_x = \frac{x^c p_x}{E} \end{cases}$$

The income effect part is $-\eta_x\theta_x$. Note that we need $x^* = x^c$ in order to have the identity $x^c(p_x, p_y, \overline{U}) = x^*(p_x, p_y, I)$ hold, so $x^* = x^c$ in the above proof.

Q6:

(1).

$$Q = \frac{10}{p} - 2 = 0 \Longrightarrow p = \frac{10}{2} = 5$$

$$CS = \int_3^5 \left(\frac{10}{p} - 2\right) dp = (10\ln 5 - 2\times 5) - (10\ln 3 - 2\times 3) = 1.108256$$

(2).

Consumer surplus is the gap between willingness to pay and the actual market price, it can be interpreted as the money saved by consumers when they pay the price which is lower than the maximum amount they are willing to pay. For example, John is willing to pay at most \$400 for a 250GB PlayStation 3 console, but the actual price he paid is \$299, so the gap, \$400 - \$299 = \$101, is John's consumer surplus.

(3).

$$CV = E\left(p_x^{old}, p_y, \overline{U}^{old}\right) - E\left(p_x^{new}, p_y, \overline{U}^{old}\right)$$

$$EV = E\left(p_x^{old}, p_y, \overline{U}^{new}\right) - E\left(p_x^{new}, p_y, \overline{U}^{new}\right)$$

To calculate CV and EV, we need to use compensated demand and expenditure function which are functions of target utility level. Which target utility to use, the one before the price change or the one after the price change, separates CV from EV.