Exam 2

You have 60 minutes for this exam. There are six questions in this exam.

Question 1: [15 points] Suppose that you know for two goods, x and y, $p_x = 1$ and $p_y = 2$, the target utility level is 30, find the **compensated demand** of x and y for the following utility functions:

1. $U(x, y) = x^{\frac{1}{4}} y^{\frac{3}{4}}$
2. $U(x, y) = 2x + 6y$
3. $U(x, y) = \min(6x, 5y)$

Question 2: [20 points] You are given the following discrete distribution of a random variable, x:

x	8	1	7	2
prob(x)	0.3	0.1	0.2	0.4

1. find the expectation of x.
2. find the variance of x.
3. find the standard deviation of x.
4. what is the expected utility if the utility function is $U(x) = \ln x^2$? Is the consumer with this utility function risk-averse, risk-neutral or risk-loving? Prove your statement.
5. what is the expected utility if the utility function is $U(x) = x^2 + 2x$? Is the consumer with this utility function risk-averse, risk-neutral or risk-loving? Prove your statement.

Question 3: [10 points] Suppose that you are deciding how to design your investment portfolio which involves two stocks, A and B. You have the following statistics available, $E(A) = 4, E(B) = 1, V(A) = 9, V(B) = 4$, and $COV(A, B) = 0$. Assume that you use mean–variance utility function, $U(x) = E(x) - \frac{1}{2}V(x)$, to represent your preference, find the optimal portfolio weights for A and B.

Question 4: [15 points] Suppose that there are three goods, x ,y, and z, their prices are p_x, p_y and p_z, respectively. $p_x = 1$, $p_y = 8$, $p_z = 2$ and total income is 100. We also know the demands are $x = 20$,$y = 5$, and $z = 20$. Income elasticities of x and y are, $\eta_x = 0.5$ and $\eta_y = 5$:

1. what is the income elasticity of z, η_z?
2. which good is luxury? which good is inferior? which is normal?

Question 5: [20 points] You are given this identity, $X^c(p_x, p_y, \bar{U}) = X^*(p_x, p_y, I)$, which is the situation where Marshallian demand equals compensated demand. Derive the Slutsky equation in elasticity form, and point out which part represents income effect.

Question 6: [20 points] Suppose that you are given this demand function, $Q = \frac{10}{p} - 2$, and you know that the market price is 3:

1. what is the value of consumer surplus?
2. what does consumer surplus represent?
3. what is the difference between compensating variation (**CV**) and equivalent variation (**EV**)?