Duopoly Models
Econ 301, Summer 2013
Iowa State University

Cournot Model:

Two identical firms compete in quantity

$$Q = q_1 + q_2 = 150 - P$$

$$MC_1 = MC_2 = 30$$

Firm 1:

$$\max_{q_1} \pi_1 = Pq_1 - MC_1 q_1 = (P - MC_1)q_1 = (150 - q_1 - q_2 - 30)q_1 = 120q_1 - q_1 q_2 - q_1^2$$

$$\frac{d\pi_1}{dq_1} = 120 - q_2 - 2q_1 = 0 \Rightarrow q_1^* = \frac{120 - q_2}{2}$$

This is the **best response function** of firm 1: his optimal quantity depends on his rival, firm 2's strategy, i.e., how much to produce, q_2. This is the **strategic interdependence** introduced in game theory!

Firm 2:

$$\max_{q_2} \pi_2 = Pq_2 - MC_2 q_2 = (P - MC_2)q_2 = (150 - q_1 - q_2 - 30)q_2 = 120q_2 - q_1 q_2 - q_2^2$$

$$\frac{d\pi_2}{dq_2} = 120 - 2q_2 - q_1 = 0 \Rightarrow q_2^* = \frac{120 - q_1}{2}$$

This is the **best response function** of firm 2: his optimal quantity depends on his rival, firm 1's strategy, i.e., how much to produce, q_1.

Nash equilibrium: the intersection of the above two best response functions, i.e., $q_1^* = q_2^*$

$$q_1^* = q_2^* \Rightarrow q_2^* = \frac{120 - q_1}{2} = \frac{120 - q_2}{2} \Rightarrow q_1^* = q_2^* = 40$$

So the *Nash equilibrium* is $[q_1^* = 40, q_2^* = 40]$

$$P = 150 - Q = 150 - q_1 - q_2 = 70$$

Profits or payoffs for each firm:

$$\pi_1^* = \pi_2^* = (70 - 30) \times 40 = 1600$$

Bertrand Model:

Two identical firms compete in price:

(1) Identical firm case:

$$Q = q_1 + q_2 = 150 - P$$

$$MC_1 = MC_2 = 30$$

In this case, the *Nash equilibrium* is for both firms to set price equal to marginal cost (which is exactly like a perfectly competitive market model),

$$P_1^* = P_2^* = 30$$

since firms are identical, they are going to equally split the market demand,

$$Q = 150 - P = 120 \Rightarrow q_1^* = q_2^* = 60$$

Profits or payoffs for each firm:

$$\pi_1^* = \pi_2^* = (30 - 30) \times 60 = 0$$

(2) Asymmetric firm case:

$$Q = q_1 + q_2 = 150 - P$$

$$\begin{cases} MC_1 = 10 \\ MC_2 = 20 \end{cases}$$

We can see that firm 1 has competitive advantage in the sense of producing at a strictly lower cost than firm 2, so the *Nash equilibrium* for this case is

$$\begin{cases} P_1^* = 20 - \epsilon \\ P_2^* = 20 \end{cases}$$

where $\epsilon > 0$ is a very small number, for example, a penny, so firm 1 can sell at a price which is lower than firm 2, he is going to take the entire market because firm 2 can not accept the price which is lower than its marginal cost:

$$\begin{cases} q_1^* = Q = 150 - P_1^* = 130 + \epsilon \\ q_2^* = 0 \end{cases}$$

Profits or payoffs for each firm:

$$\begin{cases} \pi_1^* = (P_1^* - MC_1) \times q_1^* = (10 - \epsilon) \times (130 + \epsilon) \approx 1300 \\ \pi_2^* = 0 \end{cases}$$

Stackelberg Model:

Two identical firms compete in quantity, but in a sequential-move setting: firm 1 first decides how much to produce, given firm 1's decision, firm 2 then decides how much to produce.

$$Q = q_1 + q_2 = 130 - P$$

$$MC_1 = MC_2 = 10$$

Solve this game by backward induction:

Firm 2's problem:

$$\max_{q_2} \pi_2 = (P - MC_2)q_2 = (130 - q_1 - q_2 - 10)q_2 = 120q_2 - q_1q_2 - q_2^2$$

$$\frac{d\pi_2}{dq_2} = 120 - 2q_2 - q_1 = 0 \Rightarrow q_2^* = \frac{120 - q_1}{2}$$

Firm 1's problem, knowing firm 2's best response function:

$$\max_{q_1} \pi_1 = (P - MC_1)q_1 = (130 - q_1 - q_2 - 10)q_1 = (130 - q_1 - \frac{120 - q_1}{2} - 10)q_1 = 60q_1 - \frac{1}{2}q_1^2$$

$$\frac{d\pi_1}{dq_1} = 60 - q_1 = 0 \Rightarrow q_1^* = 60 \Rightarrow q_2^* = \frac{120 - 60}{2} = 30$$

So the *Sub-game perfect equilibrium* (**SPE**) is $[q_1^* = 60, q_2^* = 30]$

$$P = 130 - Q = 130 - q_1 - q_2 = 40$$

Profits or payoffs for each firm:

$$\begin{cases} \pi_1^* = 60q_1 - \frac{1}{2}q_1^2 = 1800 \\ \pi_2^* = 120q_2 - q_1q_2 - q_2^2 = 900 \end{cases}$$

What happens if those two firms play Cournot game (simultaneous move game)?

Firm 1:

$$\max_{q_1} \pi_1 = (P - MC_1)q_1 = (130 - q_1 - q_2 - 10)q_1$$

$$\frac{d\pi_1}{dq_1} = 120 - q_2 - 2q_1 = 0 \Rightarrow q_1^* = \frac{120 - q_2}{2}$$

Firm 2:

$$\max_{q_2} \pi_2 = (P - MC_2)q_2 = (130 - q_1 - q_2 - 10)q_2$$

$$\frac{d\pi_2}{dq_2} = 120 - 2q_2 - q_1 = 0 \Rightarrow q_2^* = \frac{120 - q_1}{2}$$

Nash equilibrium: the intersection of the above two best response functions, i.e., $q_1^* = q_2^*$

$$q_1^* = q_2^* \Rightarrow q_2^* = \frac{120 - q_1}{2} = \frac{120 - q_2}{2} \Rightarrow q_1^* = q_2^* = 40$$

So the Nash equilibrium is $[q_1^* = 40, q_2^* = 40]$

$$P = 150 - Q = 130 - q_1 - q_2 = 50$$

Profits or payoffs for each firm:

$$\pi_1^* = \pi_2^* = (50 - 10) \times 40 = 1600$$

We can see that compared with Cournot case, in the Stackelberg setting, firm 1 wins because of higher profits; consumers win because of lower market price; firm 2 loses because of lower profits. In the Stackelberg model, the firm who moves first takes the majority of the market while the follower only takes the residual demand. This is called "**first-mover advantage**" in industrial organization!